RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

B.A./B.Sc. SECOND SEMESTER EXAMINATION, MAY 2014

FIRST YEAR

Date : 23/05/2014 **PHYSICS (Honours)**

Time : 11 am – 3 pm Paper : II Full Marks : 100

[Use a Separate Answer Books for each group]

Group – A

Answer **any four** questions :

1. a) Show that for odd function f(x) will contain only sine terms in its Fourier Series : [3]

   b) $f(x)=\begin{cases}1 & 0<x<\frac{1}{2}\\ 0 & \frac{1}{2}<x<1\end{cases}$

   i) Sketch the given function on (0,1) and continue it with period 1. [1]

   ii) What is the evenperiod extension of the above function, sketch it and obtain the Fourier expansion of it. [6]

2. a) Write down the Laplace's equation in spherical polar coordinate. [1]

   b) Separate out θ-part and show that it leads to Legendre equation. [4]

   c) A rectangular metal plate has two long sides and the far end at 0°C and the base at 100°C. The width of the plate is 0·1m. Find the steady state temperature distribution inside the plate. Calculate the temperature at the point (5,5). [4+1]

3. a) Consider a reference system R rotating with uniform angular velocity $\vec{\omega}$ relative to an inertial frame S, having a common origin. If $\vec{u}$ and $\vec{v}$ are the velocities of the particle, as seen from R and S, respectively, Show that $\vec{v}=\vec{u}+\vec{\omega}\times\vec{r}$, where $\vec{r}$ is the instantaneous position vector of the particle. [4]

   b) Use (a) to obtain the equation, motion of the particle in the rotating frame R, given that $\vec{F}$ is the applied force. Interpret the various terms in this expression. [6]

4. a) Set up an appropriate rotating coordinate system on the surface of the earth at any place with latitude λ.

   Assuming that $\vec{g}$ remains constant, write down the equation of motion of a projectile of mass m, with respect to this frame.

   A projectile is shot vertically upwards with constant speed u_0. Find where it will land when it reaches the ground. [1+2+3]

   b) Show that in a motion under a central force, the appropriate coordinate system to describe the motion is the plane polar (r,θ) — system. Hence show that in any central force motion, $r^2\dot{\theta}=\text{constant}$. [4]

5. a) A particle moves under a central force in an orbit given by, $r=k\theta^2$, ($k>0$, constant).

   i) Find the force

   ii) Determine how θ varies with time

   iii) Find the energy E of the particle. [4]

   b) A particle of mass m moves in an elliptic orbit under the force law, $f(r)=-\frac{k}{r^2}$, k>0.

   i) Show that the total energy E is given by $E=-\frac{k}{2a}$,where a is the semi-major axis of the ellipse. [3]

   ii) If T is the time-period for orbital motion. Show that T^2 is proportional to a^3. Explain any assumptions made. [3]

6. a) A cylinder has a mass M, length ℓ and radius r. Find the ratio of ℓ to r if the moment of inertia about an axis through the centre and perpendicular to its length is a minimum. [3]

   b) Obtain an expression for the moment of inertia of a rigid body about an axis defined by in through a given point O of the body, is terms of the components of the inertia tensor relative to same arbitrary body axes through O. [5]

   c) Define principal axes of the body, and rewrite the inertia tensor in terms of these axes. [2]

7. A rigid body is rotating with angular velocity ω about an axis through O, the origin and having direction cosines ℓ, m, n.

   a) Find an expression for the kinetic energy of rotation (T_{rot}). How is the expression modified for principal axes system? [3+2]

   b) Find an expression for angular momentum (L) of the rigid body about the axis (ℓ, m, n). How is the expression modified for principal axes system? [3+2]

<u>Group – B</u>

Answer **<u>any two</u>** questions :

8. a) Define modulus of rigidity of a solid body. Derive a relation between Young's modulus, rigidity modulus and Poisson's ratio. [2+4]

   b) Prove that torsional rigidity of a hollow cylinder is greater than that of solid cylinder having same cross-sectional area. [4]

9. a) A liquid under streamline is flowing through a horizontal capillary tube. Derive an expression for the volume of liquid flowing through it. Make necessary energy correction to this expression. [4+1]

   b) Obtain an expression for the velocity of a body under motion through a viscous liquid. Hence find its terminal velocity. [3+1]

10. a) What do you mean by angle of contact in case of liquid. Deduce an expression for the rise of liquid in a capillary tube in terms of angle of contact. [1+3]

    b) State Bernoulli's theorem. Derive this theorem from the Euler's equation of motion of a fluid. [1+5]

11. a) State and prove Gauss's theorem of gravitation. [1+3]

    b) Calculate gravitational potential and intensity at a point, distance x from the centre on the axis of a thin uniform disc of mass M and radius R. [4]

    c) Define gravitational self energy. [2]

<u>Group – C</u>

Answer **<u>any four</u>** questions :

12. a) Write down the assumptions made for the propagation of acoustic wave in solid media. [2]

    b) Prove that the velocity of sound wave in long solid media is $\sqrt{\frac{Y}{\rho}}$,where Y is the Young's modulus and ρ is the density of the metal. [5]

    c) Two sound waves one in air and other in water, having the same frequency and intensity. Calculate the ratio of the pressure amplitude of two waves. Density of air 0·00120 gm/sqcm, velocity of sound in air 331 m/sec and in water 1510 m/sec. [3]

13. a) The expression for displacement of a vibrating string of mass M and length L fixed at both ends is

    $$Y = \sum_{n=1}^{\infty} C_n \sin\frac{n\pi x}{L}\cos(\omega t - \phi_n)$$

    where C_n, ω_n and ϕ_n are amplitude at a distance x, angular frequency and internal phase of the n-th harmonics respectively. Show that the energy of the string is $E = \frac{M}{4}\sum_{n=1}^{\infty} C_n^2\omega_n^2$. [6]

    b) The wind is blowing over some suspended cables on a bridge. Causing a force that is approximately represented by $F = 0{\cdot}02 \sin 2\ell\pi t\,N$. Is resonance possible either fundamental or any harmonics, if the tension of the cable be 49000 N, the cable has a mass of 10 Kg/m and it is 10m long? [4]

14. a) A source of sound is approaching an observer with velocity u and the observer is also approaching the source with velocity v, if the emitted frequency be f, calculate the frequency of the sound as measured by the observer. Assume the wind to be stationary. [5]

b) For short wavelength water waves the phase velocity c depends on the wavelength λ according to the formula

$C = \frac{A}{\sqrt{\lambda}}$ [A being a constant] Show that group velocity $c_g = \frac{3C}{2}$. [2]

c) A particle is subjected to two simple harmonic motions at right angles to each other having the same frequency. Show that the right locus of the particle is an ellipse. [3]

15. a) Two point sources of light having a constant phase relationship between them emit light waves of same frequency which can interfere. If the intensity of the waves from the sources is the same I_0, show that on interfering they can produce a maximum intensity of $4I_0$ and a minimum of zero intensity. Derive the necessary formula. [5]

b) Explain temporal and spatial coherence of light waves. Give an account of the temporal coherence with reference to young's double-slit experiment. [3+2]

16. a) A monochromatic beam of light of wavelength λ is incident on a thin film at an angle of incidence ϕ. The film is of uniform thickness 'd' and of refractive index η. Find the condition of constructive and destructive interference between the reflected wave from the top surface and the wave coming out of the film after suffering one reflection at the bottom surface of the film. Show that the minimum intensity under destructive interference is completely zero. (You can assume the relationship between the transmission and reflection co-efficients). [5]

b) Explain how you can use Fabry-Pevot interferometer for fine structure analysis of spectral lines. [3]

c) An incident beam of light contains two close spectral lines of wave lengths λ_1 and λ_2 with mean wavelength 5168Å. Interference frings are observed in the field of view close to central region and for two successive consonances (maximum visibility) the plate separation is to be changed by 1·335mm. Calculate λ_1 and λ_2. [2]

17. a) Consider a monochromatic beam of light incident normally on a slit of width b. If he wavelength be λ and the width $b = 5\lambda$, derive analytically the intensity pattern as a function of angle in the region far from the slit. Sketch the intensity pattern and give the positions of the first two minima. Find approximately the position of the first secondary maximum and its intensity compared to the central peak. [6]

b) What is a zone plate? A source of monochromatic wave of wavelength λ is at a distance u from the plate. Find the area of the half period zones with respect to the image point at a distance v from the zone plate on the side opposite to the source. Show that the radii of different half period zones are proportional to the square root of natural numbers. Show that a zone plate behaves as a converging lens with multiple focii. [4]

18. a) Find the intensity distribution as a function of angle formed by a grating having N identical parallel slits at a large distance so that a Fraunhofer pattern is achieved. Each slit is of width ‘b’ and separated from the next by a distance ‘c’. You can assume the intensity distribution of a single slit diffraction.

Find the condition for the principal maxima.

Sketch the intensity pattern for a grating having 6 slits. [4+1+1]

b) What is Rayleigh's criterion for the resolving power of an optical instrument? Two point sources of light of wavelength λ are placed symmetrically with respect to a slit of width b. Find the angle between the sources with respect to the slit when the sources are just resolved according to Rayleigh's criterion.

The headlights of a car are 1·3 m apart. The pupil of the eye has a diameter of 4mm. The mean wavelength of light is $\lambda = 500\,\text{nm}$. Estimate the distance at which the headlights can just be resolved. [1+2+1]